Experiments on the cooling by natural convection of an array of vertical heated plates with constant heat flux

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This paper presents experimental results that aim to document the phenomenon of cooling by natural convection of an array of vertical plates with uniform and equal heat fluxes. The working fluid is air. The effect of several factors on the plate temperature distribution was determined. These factors are the spacing of the plates, the existence of a floorlike flow obstruction near the entrance of the vertical channel, and the existence of a ceilinglike flow obstruction near the exit of the vertical channel. In several cases, these factors had a paramount effect on the plate temperature distribution. Examined also was the impact on the effectiveness of the natural convection cooling of a second row of plates positioned under the row of plates under investigation in an aligned or a staggered fashion.

Keywords: *natural convection, array of heated plates, electronic equipment*

Introduction

Natural convection between heated parallel plates is an important subset of general studies involving natural convection in partially enclosed spaces. The wide range of engineering applications includes heat exchangers, passive solar heating and, perhaps most significant, the packaging of electronic equipment.

Of particular interest to this investigation are studies focusing on the phenomenon of natural convection between vertical plates with constant heat flux. Sparrow and Gregg[1] studied theoretically the problem of natural convection from a single vertical plate with constant heat flux, as well as the more general problem of natural convection from a single vertical plate with arbitrary temperature distribution.[2] The parallel plate geometry was studied theoretically by Aung[3] in the fully developed limit and by Aung *et al.*[4,5] for the developing flow configuration. In these studies[3-5] both symmetric and asymmetric heating of the plates were considered.

Experimental results for the problem of free convection between parallel plates with constant heat flux are of limited existence in the open literature. Recently, Wirtz and Stutzman[6] performed experiments in the developing temperature field regime. A correlation was reported, valid in a certain range of values of the Rayleigh number, that estimates the maximum temperature variation of the plates for a given heat flux.

Relevant to the effect of inclination on natural convection from flat surfaces with constant heat flux are the works of Vliet,[7] and Vliet and Lin.[8] Overall, our literature review revealed that the problem of natural convection from vertical plates with constant heat flux has received marginal attention compared with the problem of natural convection from isothermal vertical plates whose various aspects have been extensively investigated both theoretically and experimentally.[9-14]

A commonly found printed circuit board arrangement in the telecommunications industry places boards in a vertical, parallel fashion, which facilitates buoyancy-induced air flow for cooling. This situation is conveniently simulated with an array of heated parallel plates; this paper considers the impact of various geometric parameters on the cooling of heated parallel plates. The geometric parameters of interest are those commonly controllable in the design of electronic packaging.

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Received 13 October 1986 and accepted for publication 27 April 1987

0142-727X/87/040313-07$03.00

The experimental apparatus and procedures were based on the "type 16" printed circuit board (a telecommunications industry standard). The parallel plate dimensions were taken from the type 16 board outline, and the supporting shelf simulates the enclosure typically used to hold these boards.

The experimental investigation begins with the study of plate spacing and the corresponding impact on plate temperature rise. The investigation continues with the consideration of "floor" and "ceiling" obstructions to vertical air flow; these tests simulate arrangements with limited buoyancy-induced air flow. It is very common in the telecommunications industry to stack equipment; tests are performed to determine the impact of equipment stacking, as well as the effect of staggering cards, on convective cooling.

Experimental apparatus and procedures

The experimental apparatus was developed to provide the ability to adjust several geometric parameters that affect free convection cooling of the heated parallel plates. Specific design parameters are drawn from electronic equipment commonly found in the telecommunications industry. Although the test apparatus will only approximately model electronic equipment, the conclusions drawn from this research will be of significant value.

The heated plates are constructed as shown in Figure 1(a). A flexible heating element is placed between two aluminum plates; the aluminum plates are secured by the use of highly conductive silicone paste. The heating elements and aluminum plates are 171.5 by 305 mm; each plate is 0.75 mm thick, and the heating element is 1.15 mm thick. The conductive silicone paste adds 0.35 mm, yielding an overall thickness of 3 mm for the plate assembly. Each heater is capable of delivering up to 50 watts of power. One of the heated plates, named the thermocouple probe plate throughout this study, is used for data collection. Fine insulated thermocouple wire, 36 gauge with an overall diameter of 0.28 mm, including insulation, runs between the heating element and one of the aluminum plates. Small holes, 1.50 mm diameter, in the plate allow the wire to come through the plate and attach to the outer surface. The thermocouples are secured to the plate with highly conductive silicon paste. This arrangement of the thermocouples allows plate temperature measurements along the midheight and midlength of the plate. A total of 11 thermocouples is used to obtain plate temperature distribution information. Table 1 lists the exact distances of the thermocouples distributed vertically along the centerline of the probe plate, from the bottom of this plate.

Figure 1(b) shows the supporting shelf. It was designed to simulate the boundaries of common electronic equipment. The four vertical surfaces are solid; the sides are constructed out of steel and the front and rear surfaces out of plexiglas. The reason for using plexiglas is to provide visibility of the cards for alignment purposes. The top and bottom remain open, allowing unobstructed vertical air flow. As Figure 1(b) shows, the cards are placed vertically in the shelf in parallel fashion and run perpendicular to the plexiglas surfaces.

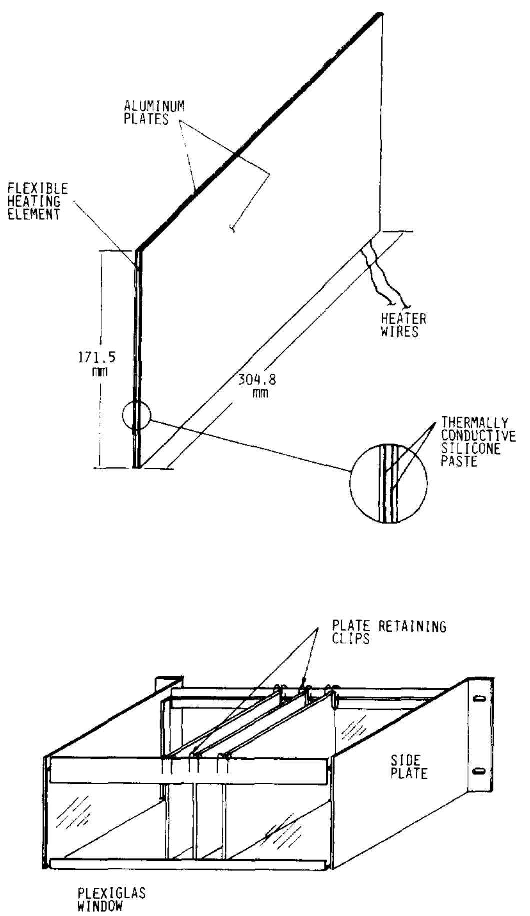

Figure 1 The experimental setup for the plate spacing tests: (a) One of the heated plates; (b) The shelf surrounding the plates

The experiments can be divided into three categories: (1) single shelf tests, (2) floor and ceiling tests, and (3) dual shelf tests. Figure 2 shows schematically the various test geometries.

In the single shelf tests, the test shelf is mounted to a vertical rack. In this test setup, the only changing geometric parameter is plate spacing. In the floor tests, a large table with an adjustable rack is used: The test shelf is secured to the adjustable rack, and the shelf is moved vertically with respect to the floor. The floor is constructed with plywood and styrofoam insulation to approximate an adiabatic surface. Combination floor and ceiling tests are performed with the addition of an adiabatic ceiling. The construction of the ceiling is similar to that of the floor, having the same perimeter and the same amount of insulation. The ceiling is supported at its four corners and is adjusted so that the shelf is centered between it and the floor. Ceiling tests are performed with the free standing rack used in the single shelf tests. The test shelf is mounted at the top of the rack, and the ceiling is supported with adjustable floor stands.

Dual shelf tests are performed to simulate the condition of having stacked equipment. The upper shelf contains the probe plate, and the lower shelf duplicates the plate arrangement of the upper shelf. The two main geometric parameters of interest are shelf spacing and card staggering. The spacing between the shelves is adjusted, and tests are performed where the heated cards are aligned or staggered by one half of the plate spacing.

A power distribution panel is secured to the rear of the rack in each of the test setups. The heating elements are wired in parallel to a variable transformer with the help of the distribution panel. A digital voltmeter is used to measure the voltage supplied to the heating elements. Thermocouple data is recorded directly in terms of temperature through the use of a temperature data logger.

A similar test procedure is followed for all the tests performed. In general, a given test begins with the alignment of the heated

Table 1 List of symbols identifying the location of the thermocouples at the centerline of the probe plate

◇	PROBE 7	157.2 mm (TOP)
×	PROBE 6	128.6 mm
▽	PROBE 5	100.0 mm
□	PROBE 4	85.7 mm
+	PROBE 3	71.4 mm
△	PROBE 2	42.9 mm
○	PROBE 1	14.3 mm (BOTTOM)

Notation

a	Plate spacing, mm
a^*	Nondimensionalized plate spacing, a/H
b	Floor spacing, mm
b^*	Nondimensional floor spacing, b/H
c	Ceiling spacing, mm
c^*	Nondimensionalized ceiling spacing, c/H
d	Spacing when both floor and ceiling are present, mm
d^*	Nondimensionalized spacing when both floor and ceiling are present, d/H
g	Gravitational acceleration, m/s^2
H	Height of the plate
k	Thermal conductivity of air, W/m°K
Nu	Nusselt number, Eq. (1)
Nu_∞	Nusselt number of a given single shelf test in the absence of floor or ceiling obstructions
$\mathrm{Nu}_{\mathrm{AL}}$	Nusselt number of a given shelf spacing test with the plates aligned
$\mathrm{Nu}_{\mathrm{FD}}$	Nusselt number for fully developed natural convection in a vertical channel, Eq. (4)
q	Power dissipation, W/m^2
Q	Total power dissipation, W
Ra	Rayleigh number, Eq. (2)
Ra_*	Rayleigh number used in the correlation for fully developed natural convection in a vertical channel, Eq. (5)
s	Shelf spacing, mm
s^*	Nondimensionalized shelf spacing, s/H
T_w	Plate temperature, °K
T	Ambient temperature, °K
T_0	Plate temperature at the channel entrance, °K
y	Vertical coordinate measuring upward from the channel entrance, mm
y^*	Nondimensionalized vertical coordinate, y/H
α	Thermal diffusivity, m^2/s
β	Coefficient of thermal expansion, °K^{-1}
ν	Kinematic viscosity, m^2/s

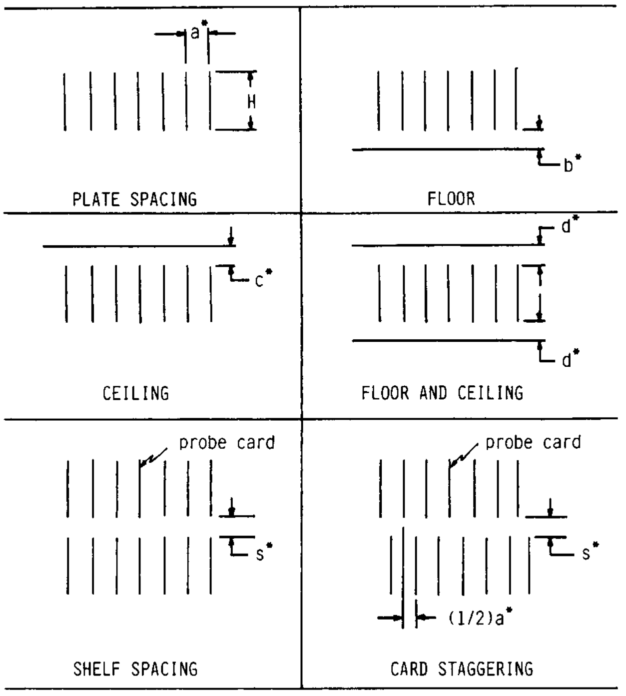

Figure 2 Schematic of the various test arrangements

plates. A plumb line is used to ensure the plates are vertical within 0.5 mm. The spacing between cards is adjusted with the use of a precision rule, ensuring parallelism within 0.2 mm. When the second shelf is being used, a plumb line is used to ensure proper card alignment between shelves. The spacing between shelves is measured at the perimeter of the shelf and is maintained within 0.5 mm. If the floor or ceiling is to be used, adjustments are made until the spacing at the perimeter of the shelf is maintained within 0.5 mm.

After the geometric parameters of the test have been fixed, power is supplied to the heating elements through the variable transformer. As the plate temperatures increase, the heater resistance can fluctuate slightly; it is necessary to readjust the voltage as the temperatures reach steady state. When steady state is reached, the temperatures are recorded on the data logger. During the course of each test, the apparatus is protected from extraneous air movement. Fluctuation in ambient air temperature is limited to 2°C per 24 hours. A typical steady state was reached within $1\frac{1}{2}$ hours.

Results and discussion

Preliminary tests were performed to determine the accuracy of the thermocouple probe plate assembly. Consideration was given to repeatability, symmetry, and continuity of the thermocouple probe measurements. To assess repeatability, arbitrary geometries with varying power level were used. Data was taken for each test geometry twice. In all cases, measurements were maintained within 0.10°C. In considering symmetry, measurements were taken when the card was in a given vertical position; the results were compared to measurements taken with the card inverted. The vertical temperature distribution was reproduced in the inverted position within 0.1°C. This is a significant test; it shows the heat supplied by the flexible heating element is being evenly distributed across the plate.

With the successful completion of the preliminary tests, it was necessary to determine the number of heated plates that would simulate an "infinite array" of vertical, parallel printed circuit boards; it is important to ensure that "end effects" are negligible. An infinite array is accurately simulated when the addition of cards no longer affects the probe plate temperature. It was determined that increasing power levels and decreasing plate spacing had the effect of increasing the number of plates required to simulate the infinite array. Tests were performed with the tightest plate spacing at the maximum power level (10.2 mm at 50 watts per card). Data was taken for one, three, five, seven, and nine plates; the probe card remaining as the center card. Comparing the seven- and nine-card tests, the maximum temperature deviation of any probe was 0.1 degrees Celsius; thus using seven cards provided accurate simulation of the infinite array. In all the following tests, the probe card remained as the center card.

The data presentation in graphical form is based on the use of certain dimensionless parameters. The local Nusselt number used to summarize the heat transfer results is defined as

$$\mathrm{Nu}=\frac{qy}{k(T_w-T_\infty)} \tag{1}$$

The variation of the local Nusselt number with the Rayleigh number as well as with several geometric parameters will provide insight into the optimization of cooling. The Rayleigh number in this study is defined as

$$\mathrm{Ra}=\frac{g\beta y^4 q}{k\nu\alpha} \tag{2}$$

As Table 1 shows, each thermocouple on the probe card is represented by a certain symbol. The same notation is used in all figures unless noted otherwise.

Figure 3(a) shows representative results of the plate spacing tests for selected values of Ra. The local Nusselt number is plotted on the vertical axis, and a^* is plotted on the horizontal axis. The results in Figure 3(a) are for a power level of 50 watts. Data points were taken for different power levels, but they are not shown here for brevity. Changing the power level in the range between 10 and 50 watts had no significant qualitative effect on the results.

Focusing on the response of an individual probe to plate spacing, we see the dependence of the local Nusselt number on plate spacing is not monotonic. Initially, the value of Nusselt increases as a^* increases indicating more effective cooling. One very interesting result occurs; there appears to be a maximum value of Nu, indicating optimum cooling at some intermediate plate spacing. Increasing the plate spacing further decreases the value of Nu. The nonmonotonic dependence of Nu on a^* is explained as follows: A narrower gap between the heated plates tends to increase the plate temperatures as well as the air flow velocity. These two opposing phenomena are responsible for the existence of a maximum in the value of Nu.

The dependence of Nu on Ra for selected values of a^* is shown in Figure 3(b). The Nusselt number varies linearly with Ra (note the logarithmic scale in the figure). It appears that the dependence of Nu on Ra is weaker for small values of a^*. The results in Figure 3(b) are for a power level of 50 watts. Other power levels yielded similar trends. In Figure 4, representative results from the present study for Nu are plotted in the same graph with results predicted by an existing theoretical correlation for fully developed natural convection flow in a channel.[6] According to the notation used in Wirtz and Stutzman,[6]

$$\mathrm{Nu_{FD}}=0.144\frac{H}{y}\mathrm{Ra}_*^{0.5} \tag{3}$$

where

$$\mathrm{Nu_{FD}}=\frac{qa}{k(T_w-T_0)} \tag{4}$$

and

$$\mathrm{Ra}_*=\frac{g\beta a^5 g}{k\nu\alpha H} \tag{5}$$

Figure 4 shows the present results and correlation (Eq. 4)

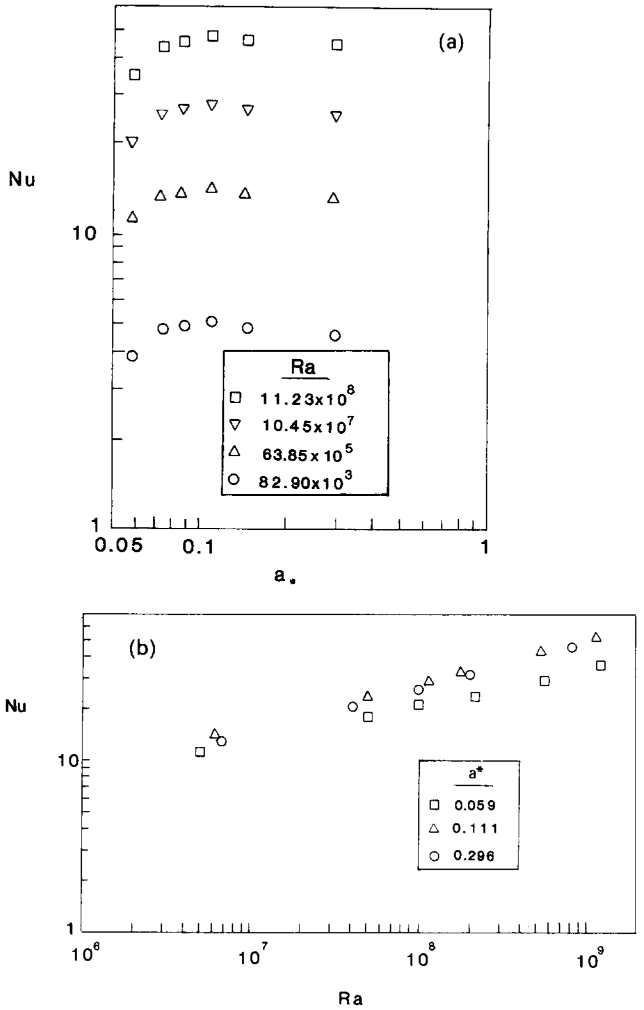

Figure 3 (a) The dependence of Nu on a^* for the plate spacing tests for Q=50 w; (b) The dependence of Nu on Ra for the plate spacing tests for Q=50 w

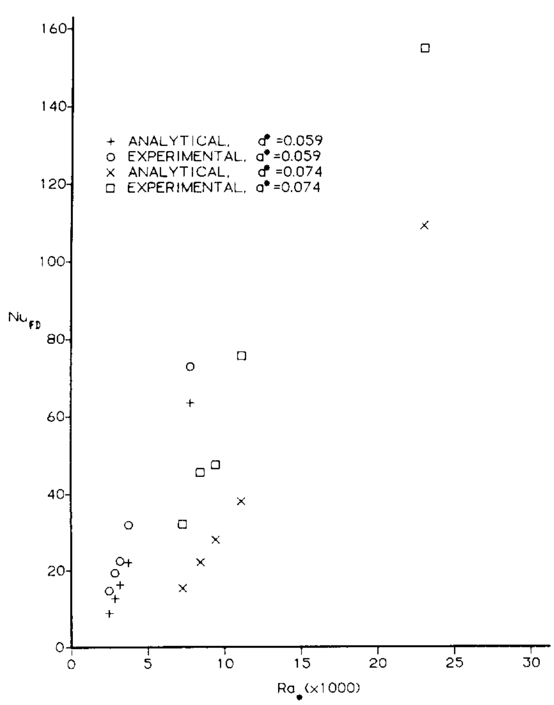

Figure 4 Comparison between present results and results predicted by a theoretical correlation for fully developed natural convection in a vertical channel

compare well for small plate spacings and far away from the channel entrance. As the plate spacing increases or as the channel entrance is approached, the agreement deteriorates. This result makes sense physically: Since Eq. (4) pertains to fully developed flow, it is expected not to hold in the developing near-entrance region. In addition, increasing the plate spacing (a^*) tends to extend the length of the developing region. The comparison in Figure 4 adds credibility to these results and proves the realistic geometries simulated in this study do not feature completely fully developed temperature fields even at the smallest value of the plate spacings ($a^* = 0.059$).

Figure 5 shows the floor spacing test results. The drastic dependence of plate temperature on floor spacing is clearly demonstrated with a comparison of the local Nusselt numbers. As the floor spacing increases, the data should approach the results of the single shelf tests ($\mathrm{Nu}/\mathrm{Nu}_\infty \rightarrow 1$). The graphs show this is indeed the result. The benefit of increased floor spacing decreases dramatically as floor spacing increases. After a certain "critical" value of b^* ($b^* \approx 0.15$ in Figure 5), nearly all the potential benefit has been obtained.

As the floor spacing increases, the data for the individual probes exhibit a tighter grouping. This indicates the temperature distribution with height is more significant when the floor is closer to the bottom of the cards. The effect of the floor is more detrimental at the bottom of the card than at the top.

Figures 5(a) and 5(b) compare the results for two plate spacings at the same power level. Figure 5(a) is for $a^* = 0.059$, whereas Figure 5(b) is for $a^* = 0.148$. The noticeable difference

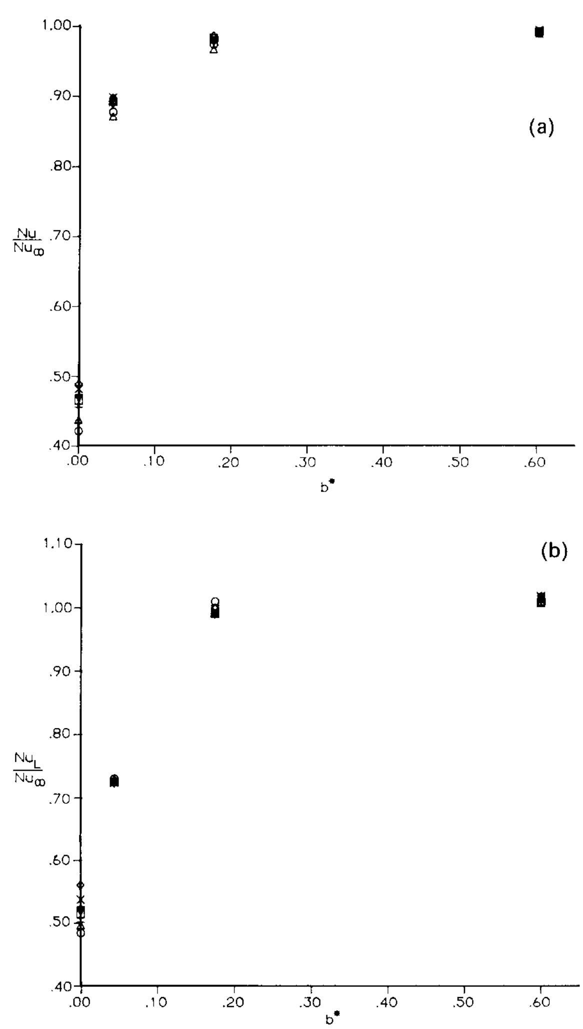

Figure 5 Floor tests: the effect of b^* on Nu (a) $a^* = 0.059$, Q=20 w; (b) $a^* = 0.148$, Q=20 w

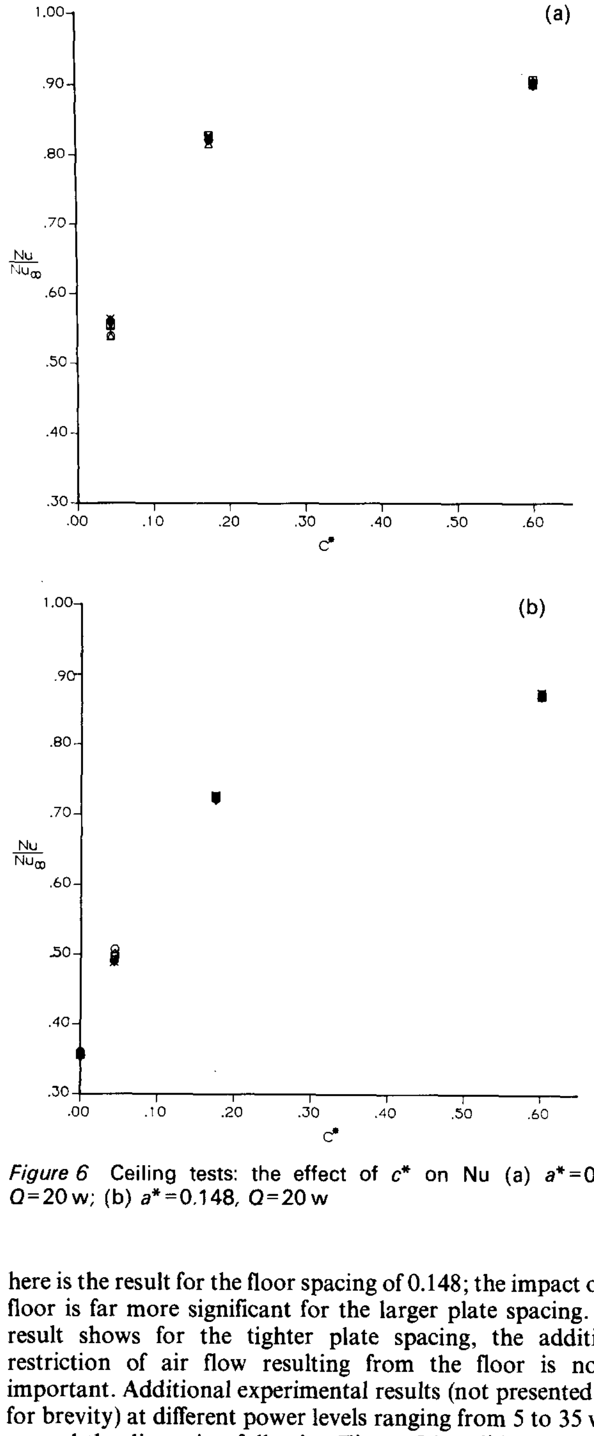

Figure 6 Ceiling tests: the effect of c^* on Nu (a) $a^*=0.059$, $Q=20$ w; (b) $a^*=0.148$, $Q=20$ w

here is the result for the floor spacing of 0.148; the impact of the floor is far more significant for the larger plate spacing. This result shows for the tighter plate spacing, the additional restriction of air flow resulting from the floor is not as important. Additional experimental results (not presented here for brevity) at different power levels ranging from 5 to 35 watts proved the discussion following Figure 5 is valid regardless of the level of power dissipation (for the range of parameters examined here).

Figure 6 shows the results for the ceiling tests. The same test parameters are used as in the floor tests, and the data are presented in the same format. As with the floor tests, increasing spacing causes the local Nusselt numbers to approach gradually the values found in the single shelf tests. Comparing Figure 6(a) with 6(b), we can deduce what the impact of plate spacing (a^*) on Nu is for the ceiling tests. As with the floor tests, the impact of the ceiling is more drastic at the larger plate spacings.

Comparing the ceiling tests (Figure 6) with the floor tests (Figure 5), we concluded the ceiling has a greater negative effect on cooling than does the floor. Although both offer a similar restriction to air flow, the ceiling has the additional effect of "trapping" the heated air. It takes larger spacing (c^*) to approach the single shelf test results with the ceiling than it does with the floor.

In the combination floor and ceiling tests, shown in Figure 7, the effects are similar to those for the individual floor and ceiling tests. As anticipated, the combined effects have a greater negative impact on cooling. One interesting result occurs; the ceiling tests and the combined tests both appear to approach the single shelf limit at the same rate. This indicates the addition of a floor obstruction is not nearly as significant if a ceiling obstruction already exists.

Figures 8 and 9 display the results of the shelf spacing tests for the aligned and the staggered configuration, respectively (Figure 2). Focusing on the aligned configuration first, we observe the value of the group Nu/Nu_∞, used to report the results so that a direct comparison with the single shelf configuration is possible, is between 0.5 and 0.6. This suggests the presence of the lower shelf reduces the cooling effectiveness of the natural convection mechanism nearly to half as much. We anticipated the existence of a lower shelf would reduce the value of Nu. However, the magnitude of this reduction is rather surprising. We also observe the actual distance between the two shelves does not have a considerable impact on the value of Nu.

As the plate spacing in each individual shelf increases—going from $a^*=0.059$ in Figure 8(a) to $a^*=0.148$ in Figure 8(b)—the value of Nu increases considerably, indicating better cooling: Larger plate spacing allows cooler air to enter the bottom of the top shelf.

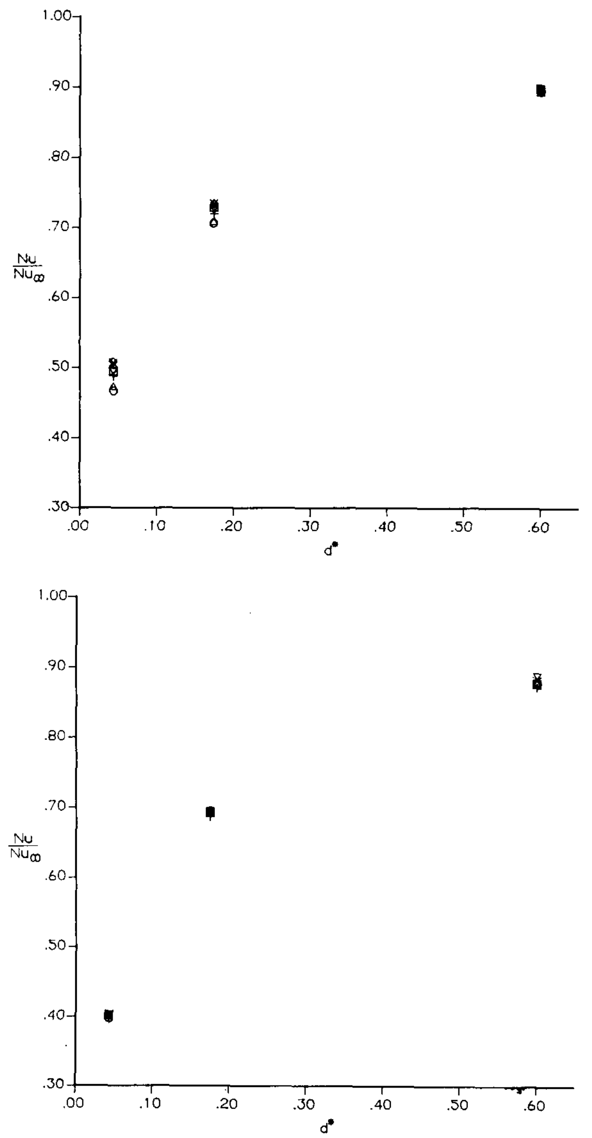

Figure 7 Floor and ceiling tests: the effect of d^* on Nu (a) $a^*=0.059$, $Q=20$ w; (b) $a^*=0.148$, $Q=20$ w

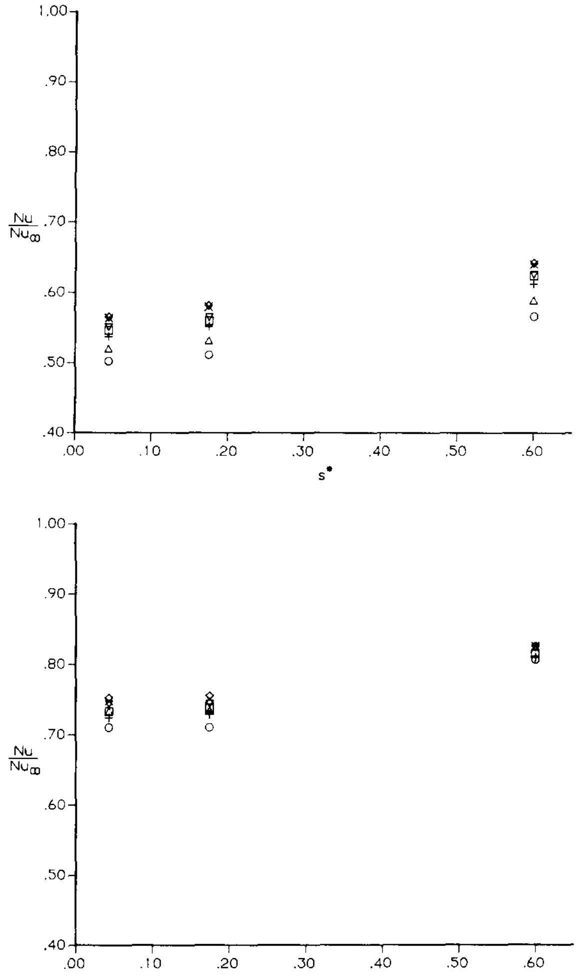

Figure 8 Shelf spacing tests: the effect of s^* on Nu (a) $a^*=0.059$, $Q=20$ w; (b) $a^*=0.148$, $Q=20$ w

The distance between shelves does not have a considerable impact on the change in Nusselt numbers. This indicates a lack of cool air entry between shelves. Most of the rising heated air from the lower shelf will pass through the upper shelf even at the larger shelf spacings. This result is very important in applications where similar equipment shelves will be mounted in a given rack. Providing space between shelves is not likely to offer appreciable benefit in cooling, and could result in an unnecessary decrease in equipment density.

Next, we turn our attention to the results of the card staggering tests (Figure 9). For these tests, the local Nusselt number is divided by the corresponding local Nusselt number of the aligned shelf tests. This provides direct information about the advantage of staggering the cards. Figure 9(a) represents a geometry in which card staggering offers no benefit to the convective cooling phenomenon. There is actually a decrease in convective cooling of the upper cards. As plate spacing increases, the benefit becomes significant, as Figure 9(b) shows. This is an important finding, for it indicates staggering is not recommended at small plate spacings. The improvement of convective cooling with card staggering diminishes with increasing shelf spacings. A similar result was found by Sparrow and Prakash,[13] who studied numerically the phenomenon of natural convection from a staggered array.

Figure 10 provides an overall view of most of the experimental data: For several geometries, the local Nusselt number of the uppermost probe is plotted against three different plate spacings. This Nusselt number corresponds to the maximum plate temperature. Each symbol represents a unqiue geometry.

This figure clearly shows the impact of increasing the card spacing is to increase the local convective cooling (reduce the maximum temperature). This result is consistent regardless of the test geometry. The tighter plate spacing produces results with a narrower band of local Nusselt numbers. This indicates the various geometries have a more significant impact on convective cooling in the case of the larger plate spacings. The

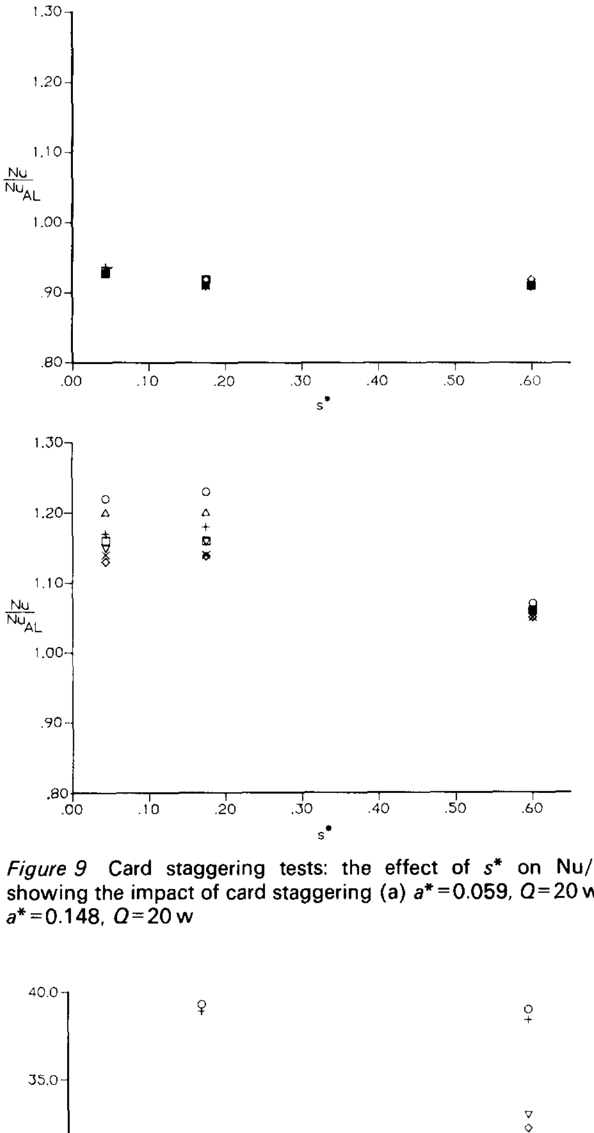

Figure 9 Card staggering tests: the effect of s^* on Nu/Nu_{AL} showing the impact of card staggering (a) $a^*=0.059$, $Q=20$ w; (b) $a^*=0.148$, $Q=20$ w

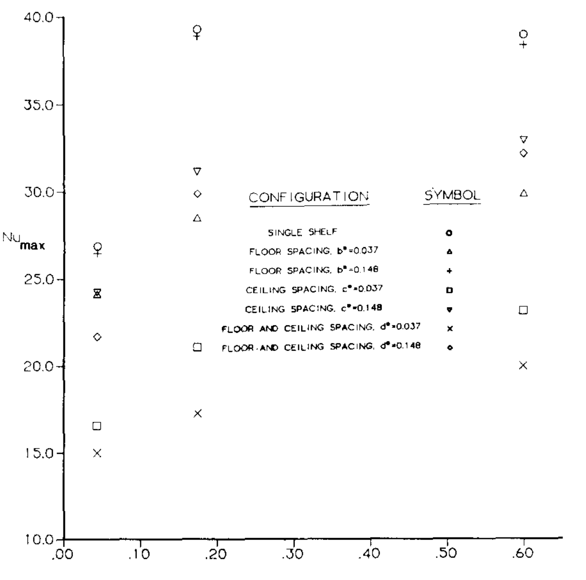

Figure 10 The dependence of the maximum Nusselt number (Nu_{max}) on various geometric parameters for the test arrangements ($Q=20$ w)

configuration with the highest Nusselt number is that with no restriction to vertical air flow. The lowest Nusselt number is obtained with the floor and ceiling very close to the shelf. Perhaps the most significant observation is the ceiling has considerably stronger "quenching" impact on convective cooling in comparison to the floor obstruction. There is also minimal difference between the ceiling test results and the floor and ceiling test results; this is particularly true for the larger floor and ceiling spacings.

Conclusions

The experimental investigations have provided many significant results that can be directly applied to the rack layout of electronic equipment. Although the heated plate does not duplicate a printed circuit board, the results are very useful for providing engineering estimates vital in the design process.

A significant result of the plate spacing tests is the linear logarithmic relationship between the local Nusselt number and the Rayleigh number for fixed values of a^*. We also determined that increasing plate spacing does not necessarily increase the convection coefficient, which implies an "optimum" plate spacing for maximizing heat dissipation.

The floor spacing tests revealed that increasing the floor spacing beyond a critical value drastically reduces the effect of the presence of the floor on the convective cooling. This result can be directly applied to rack layout of electronic equipment. It is desirable to maximize the density of racked equipment, and thus minimize any unused mounting space. If a unit requiring buoyancy-induced air flow for cooling is placed above a unit with solid top surface (a "floor"), increasing the space between units will *not* necessarily offer a significant enhancement in heat transfer.

The ceiling was shown to have a greater negative effect on cooling than the floor. The application here is quite clear; solid surfaces should remain below units requiring vertical air flow when possible. The combination floor and ceiling tests revealed the addition of a floor obstruction is not nearly as significant if a ceiling obstruction already exists.

One significant result of the shelf spacing tests is the distance between the shelves does not have a considerable impact on the change in the Nusselt numbers. This indicates a small increase in spacing between shelves will not offer an appreciable improvement in the cooling of the upper shelf. Card staggering offers more significant benefit for large card spacing and reduced distance between shelves.

Using reliable, low-cost natural convection cooling can pose a more significant design challenge than implementing more sophisticated cooling techniques. That more data does not exist to aid in the development of enhanced free convection cooling methods is unfortunate. The conclusions drawn from this research provide a general look at the impact of various geometries on free convection cooling of vertical parallel plates. Each area of general experimentation could be pursued in much greater detail to provide a broad base of data for applications in electronic packaging.

Acknowledgment

The authors gratefully acknowledge the financial support provided by Tellabs Inc. with which the experimental apparatus was constructed and instrumented. Partial support from NSF grant no. CBT 8451144 is also acknowledged.

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